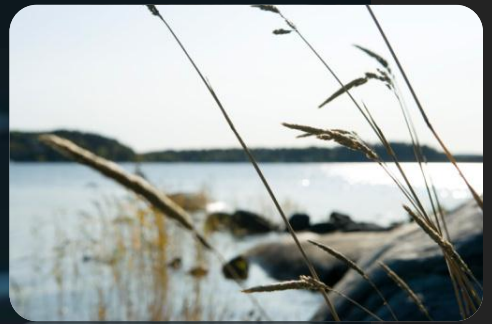


# Electric Potential

Ali Alzahrani

# Electric Potential

- **Electric potential energy** is the energy needed to change the spatial configuration of charged particles from initial to final destinations.

$$\Delta U = U_f - U_i = -W$$

- We used to take the energy at infinity is zero, therefore, the energy required to move any charged particle to any position is

$$U = -W$$

- **Electric potential 'potential'** is defined as the potential energy per unit charge, such as

$$V = \frac{U}{q}$$

# Electric Potential

- **The potential difference** between any two points is defined as

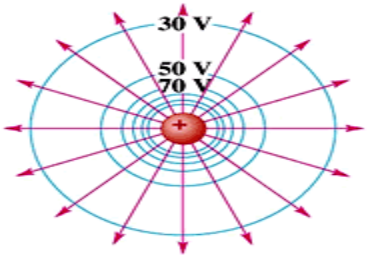
$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} = -\frac{W}{q}$$

- The SI unit of potential is '**volt** (V)' which is also J/C. However, the potential energy is measured by '**electron-volt** (eV)' which is $1.6 \times 10^{-19}$J.

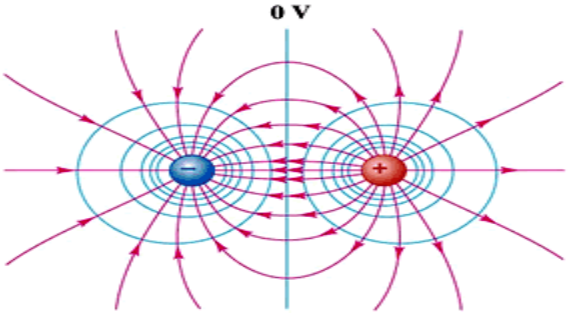
$$1 \text{ eV} = 1.6\times 10^{-19} \text{ C. } (1 \text{ J/C}) = 1.6\times 10^{-19} \text{ J}$$

- **Equipotential surfaces** are those where each point on them have the same electric potential or the potential difference between any two points is **zero**. Therefore **no net work** should be done on a charged particle to move between any two points on that surface.

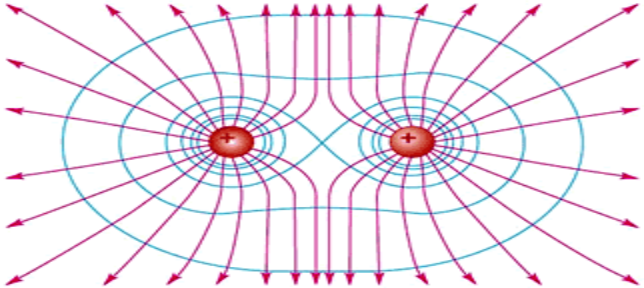
# EQUIPOTENTIAL SURFACE - EXAMPLES



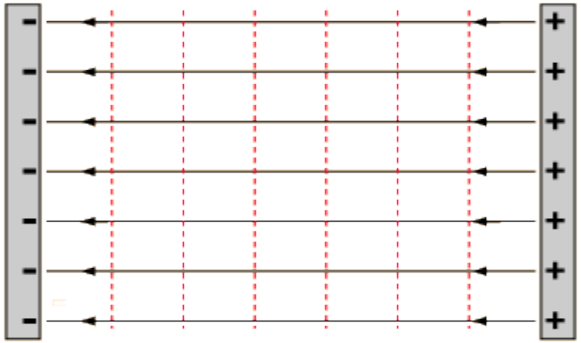

(a) A single positive charge




(b) An electric diploe



(c) Two positive charges



(d) Uniform electric field

# Electric Potential and Field

- **The electric potential** is defined in terms of **electric field** as

$$\Delta V = -\int_i^f \vec{E}.d\vec{s}$$

From this we note that 1 V = 1 N.m/C or **1 N/C = 1 V/m**. If the initial potential is zero, the above equation becomes

$$V = -\int_i^f \vec{E}.d\vec{s}$$

**NOTE**: It is important to note that the potential is a **scalar** quantity NOT a vector.

# Electric Potential due to a Point Charge

- The electric potential of a point charge $q$ at any distance $r$ can be evaluated from

$$V = -\int_i^f \vec{E}.d\vec{s}$$

Having known that the electric field due to any point charge is $E = \frac{kq}{r^2}$ we get

$$V = -\int_\infty^r \frac{kq}{r^2} dr = \frac{kq}{r}$$

**NOTE**: Positive charges produce positive potential and vice versa.

# Electric Potential due to a Group of Point Charges

- The electric potential of a collection of point charge $q_i$ at any distance $r_i$ is the summation of all potentials of each individual point charge

$$V = \sum_{i=1}^{n} V_i = k \sum_{i=1}^{n} \frac{q_i}{r_i}$$

# Electric Potential Due to a Dipole

- The electric potential due to an electric dipole $p$ at any point at distance $r$ and makes an angle $\theta$ with the axes of the dipole is defined as

$$V = \frac{k\,p\,\cos\theta}{r^2}$$

**NOTE**: The angle $\theta$ is measured from the direction of the dipole.

# Electric Potential due to a Conducting Sphere

- The electric potential of a conducting sphere of radius $R$ and charge $q$ at any distance $r$ **outside** the sphere, where the electric field (outside) is $E = \frac{kq}{r^2}$ , is

$$V = -\int_{\infty}^{r} \frac{kq}{r^2} dr = \frac{kq}{r}$$

- The electric potential of an insulating sphere of radius $R$ and charge $q$ at any distance $r$ **inside** the sphere, where the electric field (outside) is $E = \frac{kq}{r^2}$ and (inside) $E = 0$, is

$$V = -\int_{\infty}^{R} \frac{kq}{r^2} dr - \int_{R}^{r} 0 \, dr = \frac{kq}{R}$$

Therefore the electric potential at any point of the isolated conductor is invariant, leading to that no extra work is needed to carry a charge from surface to any point within the conductor, *see example* 7.

# Electric Potential due to an Insulating Sphere

- The electric potential of an insulating sphere of radius $R$ and charge $q$ at any distance $r$ **outside** the sphere, where the electric field (outside) is $E = \frac{kq}{r^2}$ , is

$$V = -\int_{\infty}^{r} \frac{kq}{r^2} dr = \frac{kq}{r}$$

- The electric potential of a conducting sphere or radius $R$ and charge $q$ at any distance $r$ **inside** the sphere, where the electric field (outside) is $E = \frac{kq}{r^2}$ and (inside) $E = \frac{kq\,r}{R^3}$, is

$$V = -\int_{\infty}^{R} \frac{kq}{r^2} dr - \int_{R}^{r} \frac{kq\,r}{R^3} dr = \frac{kq}{R}\left[1 - \frac{r^2 - R^2}{2R^2}\right]$$

# Potential Energy of a System of Point Charges

- The electric potential energy of a system of point charges $q_i$ separated by distances $r_i$ is the summation of all potential energy of these point charges

$$U = \sum_{i,j=1}^{n} U_{ij} = k \sum_{i,j=1}^{n} \frac{q_i q_j}{r_{ij}}$$

where $r_{ij}$ is the distance between charges $q_i$ and $q_j$

# E DERIVED FROM V

- The electric field can be evaluated from the expression of the potential such as

$$E = -\frac{\partial V}{\partial s}$$

In our three-dimensional space, the above definition can be rewritten as

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

Therefore the electric field vector is

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k}$$

# Worked Exercises

**1. Calculate the electric potential at a distance 5 cm from a point charge 2.5 nC.**

**Solution**

The magnitude of the electric potential at any distance from a point charge is given by

$$V = \frac{k\,q}{r}$$

Hence the electric potential at 5 cm from the charge is

$$V = \frac{k\,q}{r} = \frac{9\times10^{9}\times2.5\times10^{-9}}{0.05} = 450\ \mathrm{V}$$

# Worked Exercises

**2. A point charge produces an electric field of 180 N/C at 2 cm. Calculate the electric potential at 4 cm from the charge.**

**Solution**

The magnitude of the electric field due to a point charge is given by

$$E = \frac{k\,|q|}{r^2}$$

Therefore to find the electric potential at any point, we have to determine the charge, firstly.

$$q = \frac{E\,r^2}{k} = \frac{180 \times 0.02^2}{9 \times 10^9} = 8 \times 10^{-12}\text{C}$$

Hence the electric potential at 4 cm from the charge is

$$V = \frac{k\,q}{r} = \frac{9 \times 10^9 \times 8 \times 10^{-12}}{0.04} = 1.8\ \text{V}$$

# Worked Exercises

**3. As shown in the arrangement below, calculate the electric potential at point A. Calculate the work needed to bring a charge Q=6 nC from infinity to point A.**

**Solution**

- The electric potential of a collection of points is

$$V = V_1 + V_2 = \frac{k\,q_1}{r_1} + \frac{k\,q_2}{r_2} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = 9 \times 10^9 \times \left(\frac{3 \times 10^{-6}}{2} + \frac{-4 \times 10^{-6}}{5}\right) = 6300\ \mathrm{V}$$

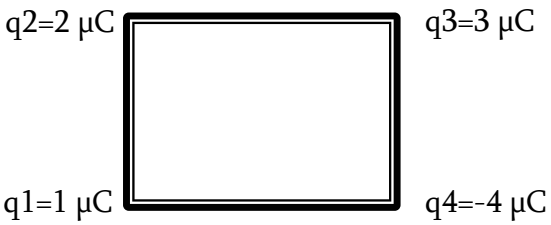
Therefore the work needed to bring a charge from infinity to this point is

$$W = -QV = -6 \times 10^{-9} \times 6300 = -3.78 \times 10^{-5}\mathrm{J} = -2.36 \times 10^{14}\ \mathrm{eV}$$

# Worked Exercises

**4. Four charges 1, 2, 3 and -4 μC are located at the corners of a square of side 3 m as shown in the figure. Calculate the electric potential energy of the system.**

**Solution**




The potential energy is

$$U = \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_3q_4}{r_{34}} + \frac{kq_4q_1}{r_{41}} = \frac{k}{r}(q_1q_2 + q_2q_3 + q_3q_4 + q_4q_1) = \frac{9 \times 10^9}{3}(2 + 6 - 12 - 4) \times 10^{-12} = -24 \text{ mJ}$$

# Worked Exercises

**5. Two equal and opposite charges 6.0 μC and -6.0 μC are separated by a distance of 2 cm. What is the electric potential at a point 30 cm away from the axis of the dipole?**

**Solution**

The electric potential due to a dipole is

$$V = \frac{k\,p\,\cos\theta}{r^2}$$

However along the axis of the dipole the angle is zero, ($\cos\theta = 1$), therefore

$$V = \frac{k\,p}{r^2} = \frac{k\,qd}{r^2} = \frac{9 \times 10^9 \times 6 \times 10^{-6} \times 0.02}{0.3^2} = 12\ \text{kV}$$

# Worked Exercises

**6. A metallic sphere of radius 10 cm has a charge of 5.0 μC. What is the electric potential at (i) 5 cm and (ii) 15 cm from its center?**

**Solution**

(i) The electric potential at any point inside a conducting sphere is

$$V = \frac{k\,q}{R} = \frac{9 \times 10^{9} \times 5 \times 10^{-6}}{0.1} = 450000 \text{ V}$$

(ii) The electric potential at any point outside a conducting sphere is

$$V = \frac{k\,q}{r} = \frac{9 \times 10^{9} \times 5 \times 10^{-6}}{0.15} = 300000 \text{ V}$$

# Worked Exercises

**7. A conducting sphere of radius 10 cm has a charge of 5.0 μC. Calculate the work required to carry a charge 2.0 μC from the surface to the center of the sphere.**

**Solution**

We know that the electric potential at any point inside a conducting sphere is

$$V = \frac{k\,q}{R}$$

Also the electric potential at the surface is

$$V = \frac{k\,q}{R}$$

Therefore the potential difference is **zero**, leading the work is **zero**, too, according to $W = -Q\,\Delta V$

# Worked Exercises

**8. At a certain region, the electric potential is found to be $V(x, y, z) = 3x^2y + z$. Calculate the electric field at point (1,0,1).**

**Solution**

The electric field relates to potential through

$$E_x = -\frac{\partial V}{\partial x} = -6xy \qquad E_y = -\frac{\partial V}{\partial y} = -3x^2 \qquad E_z = -\frac{\partial V}{\partial z} = -1$$

Therefore at the point (1,0,1) we get

$$E_x = 0 \qquad E_y = -3 \qquad E_z = -1$$

Hence the electric field is

$$\vec{E} = -3\hat{j} - \hat{k}$$